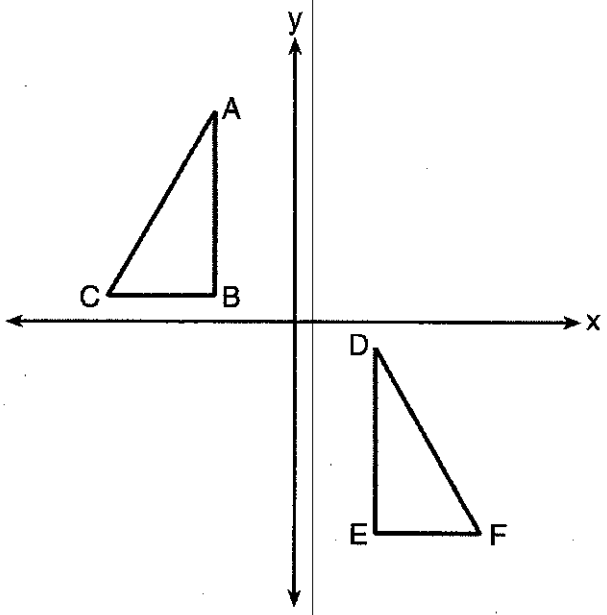
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, $\triangle ABC \cong \triangle DEF$.



Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

(1) a reflection over the x-axis followed by a translation

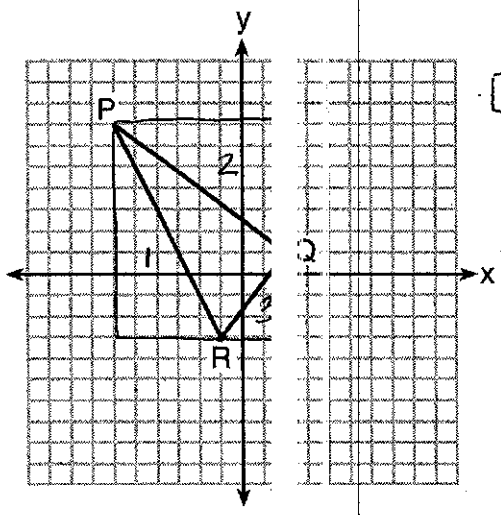
(2) a reflection over the y-axis followed by a translation

(3) a rotation of 180° about the origin followed by a translation

(4) a counterclockwise rotation of 90° about the origin followed by a translation

Use this space for computations.

2 On the set of axes below, the vert es of $\triangle PQR$ have coordinates $P(-6,7)$, $Q(2,1)$, and $R(-1,-3)$.



$\square = 10(8) = 80$

$\Delta 1 = \frac{1}{2}(10)(5) = 25$

$\Delta 2 = \frac{1}{2}(8)(6) = 24$ } 55

$\Delta 3 = \frac{1}{2}(4)(3) = 6$

$80 - 55 = 25$

What is the area of $\triangle PQR$?

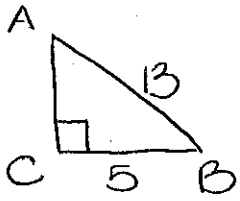
(1) 10 (3) 25

(2) 20 (4) 50

3 In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

(1) $\tan A$ (3) $\sin A$

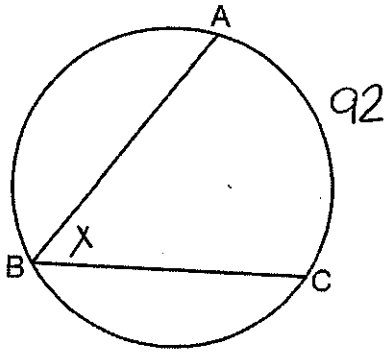
(2) $\tan B$ (4) $\sin B$



Use this space for computations.

4 In the diagram below, $m\overset{\frown}{ABC} = 268°$.

360-268=92



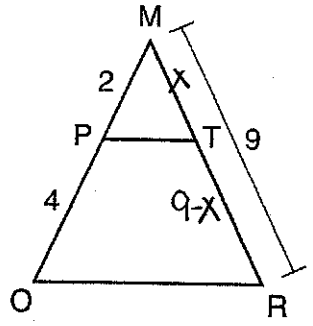
X = 92/2

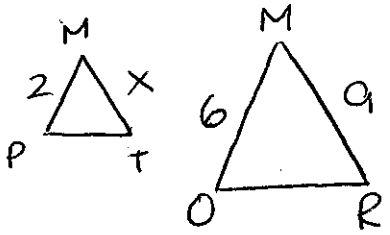
X = 46

What is the number of degrees in the measure of $\angle ABC$?

(1) 134°
(2) 92°
(3) 68°
(4) 46°

5 Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.





2/6 = X/9

6X = 18

X = 3

What is the length of $\overline{TR}$?

(1) 4.5
(2) 5
(3) 3
(4) 6

Use this space for computations.

6 A line segment is dilated by a sca factor of 2 centered at a point not on the line segment. Which sta ment regarding the relationship between the given line segment a its image is true?

(1) The line segments are perper icular, and the image is one-half of the length of the given line egment.

(2) The line segments are perpen icular, and the image is twice the length of the given line segm t.

● The line segments are paralle and the image is twice the length of the given line segment.

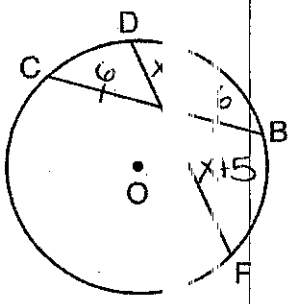
(4) The line segments are paralle and the image is one-half of the length of the given line segm t.

7 Which figure always has exactly f r lines of reflection that map the figure onto itself?

● square (3 regular octagon

(2) rectangle (4 equilateral triangle

8 In the diagram below of circle O chord $\overline{DF}$ bisects chord $\overline{BC}$ at E.



If $BC = 12$ and FE is 5 more tha DE, then FE is

(1) 13 (3 6

● 9 (4 4

$X(X+5) = 6(6)$

$X^2 + 5X = 36$

$X^2 + 5X - 36 = 0$

$(X+9)(X-4) = 0$

$X = -9 \mid X = 4$

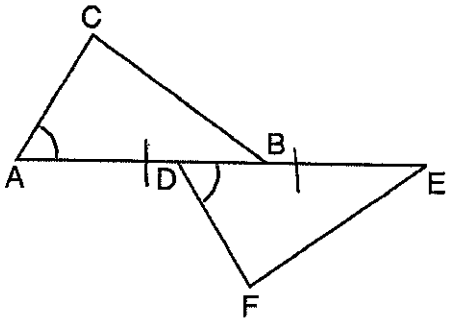
$FE = X + 5$

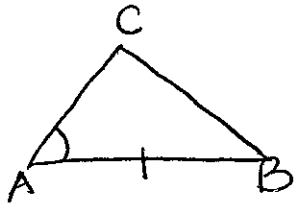
$4 + 5$

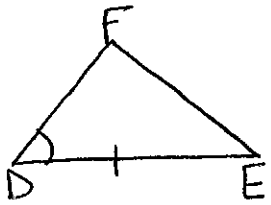
9

Use this space for computations.

9 Kelly is completing a proof based on the figure below.







She was given that $\angle A \cong \angle EDF$, and has already proven $\overline{AB} \cong \overline{DE}$. Which pair of corresponding parts and triangle congruency method would *not* prove $\triangle ABC \cong \triangle DEF$?

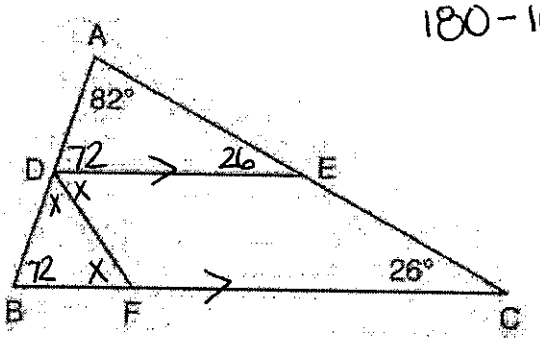
(1) $\overline{AC} \cong \overline{DF}$ and SAS

(2) $\overline{BC} \cong \overline{EF}$ and SAS

(3) $\angle C \cong \angle F$ and AAS

(4) $\angle CBA \cong \angle FED$ and ASA

10 In the diagram below, $\overline{DE}$ divides $\overline{AB}$ and $\overline{AC}$ proportionally, $m\angle C = 26°$, $m\angle A = 82°$, and $\overline{DF}$ bisects $\angle BDE$.



180 - 108 = 72

2X + 72 = 180

2X = 108

X = 54

The measure of angle DFB is

(1) 36°

(2) 54°

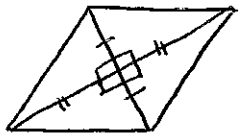
(3) 72°

(4) 82°

Use this space for computations.

11 Which set of statements would describe a parallelogram that can always be classified as a rhombus?

I. Diagonals are perpendicular bisectors of each other.
II. Diagonals bisect the angles from which they are drawn.
III. Diagonals form four congruent isosceles right triangles.



(1) I and II
(2) I and III
(3) II and III
(4) I, II, and III

12 The equation of a circle is $x^2 + y^2 - 12y + 20 = 0$. What are the coordinates of the center and the length of the radius of the circle?

$\left(\frac{B}{2}\right)^2 = \left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$x^2 + y^2 - 12y + 36 = -20 + 36$

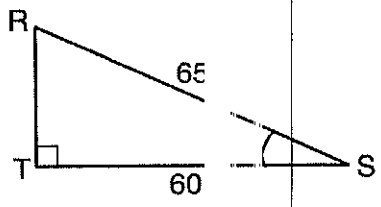
$x^2 + (y-6)^2 = 16$

$C = (0,6) \quad r^2 = 16$

$r = 4$

(1) center (0,6) and radius 4
(2) center (0,−6) and radius 4
(3) center (0,6) and radius 16
(4) center (0,−6) and radius 16

13 In the diagram of $\triangle RST$ below, $m\angle T = 90°$, $RS = 65$, and $ST = 60$.



$\cos X = \frac{60}{65}$

$X = 23$

What is the measure of $\angle S$, to the *nearest degree*?

(1) 23°
(2) 43°
(3) 47°
(4) 67°

Use this space for computations.

14 Triangle $A'B'C'$ is the image of $\triangle ABC$ after a dilation followed by a translation.

Which statement(s) would always be true with respect to this sequence of transformations?

I. $\triangle ABC \cong \triangle A'B'C'$
II. $\triangle ABC \sim \triangle A'B'C'$
III. $\overline{AB} \parallel \overline{A'B'}$
IV. $AA' = BB'$

● II, only
(2) I and II
● II and III
(4) II, III, and IV

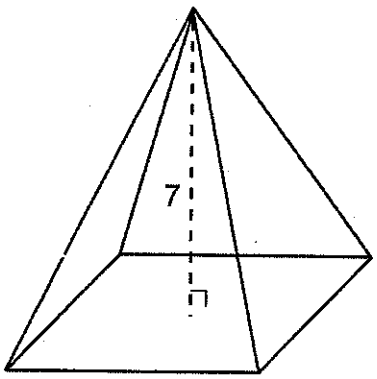
15 Line segment RW has endpoints $R(-4,5)$ and $W(6,20)$. Point P is on $\overline{RW}$ such that $RP:PW$ is 2:3. What are the coordinates of point P?

(1) (2,9)
● (0,11)
(3) (2,14)
(4) (10,2)

$R(-4,5) \xrightarrow{T\langle 10,15\rangle} W(6,20)$
$10\left(\frac{2}{5}\right)\ 15\left(\frac{2}{5}\right)$

$R(-4,5) \xrightarrow{T\langle 4,6\rangle} P(0,11)$

16 The pyramid shown below has a square base, a height of 7, and a volume of 84.



What is the length of the side of the base?

● 6
(2) 12
(3) 18
(4) 36

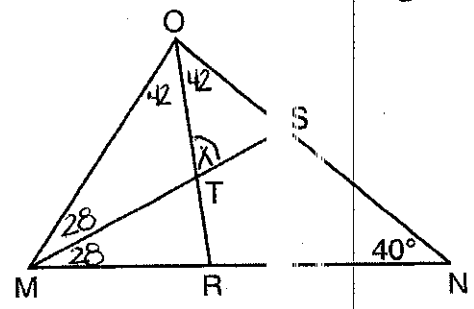
$V = \frac{1}{3}Bh$
$84 = \frac{1}{3}x(x)(7)$
$\frac{84}{1} = \frac{7x^2}{3}$
$\frac{252}{7} = \frac{7x^2}{7}$
$\sqrt{36} = \sqrt{x^2}$
$6 = x$

Use this space for computations.

17 In the diagram below of triangle MNO, $\angle M$ and $\angle O$ are bisected by $\overline{MS}$ and $\overline{OR}$, respectively. Segments MS and OR intersect at T, and $m\angle N = 40°$.

180-28-28-40 = 84/2 = 42

28+42 = 70



If $m\angle TMR = 28°$, the measure of angle OTS is

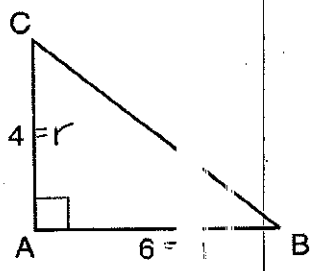
(1) 40°
(2) 50°
(3) 60°
(4) 70°

18 In the diagram below, right triangle ABC has legs whose lengths are 4 and 6.

$V = \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi(4)^2(6)$

32π



What is the volume of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

(1) 32π
(2) 48π
(3) 96π
(4) 144π

Use this space for computations.

19 What is an equation of a line that is perpendicular to the line whose equation is $2y = 3x - 10$ and passes through $(-6,1)$?

(1) $y = -\frac{2}{3}x - 5$

(3) $y = \frac{2}{3}x + 1$

● $y = -\frac{2}{3}x - 3$

(4) $y = \frac{2}{3}x + 10$

$\frac{2y}{2} = \frac{3x-10}{2}$

$y = \frac{3}{2}x - 5$

$\perp m = -\frac{2}{3}$ pt $(-6,1)$

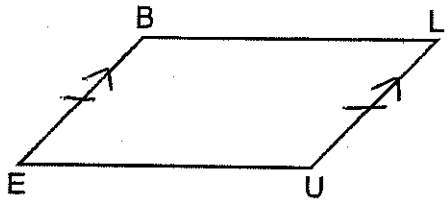
$y - y = m(x - x)$

$y - 1 = -\frac{2}{3}(x + 6)$

$y - 1 = -\frac{2}{3}x - 4$

$y = -\frac{2}{3}x - 3$

20 In quadrilateral *BLUE* shown below, $\overline{BE} \cong \overline{UL}$.



Which information would be sufficient to prove quadrilateral *BLUE* is a parallelogram?

(1) $\overline{BL} \parallel \overline{EU}$

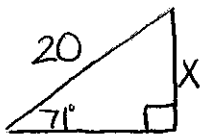
(3) $\overline{BE} \cong \overline{BL}$

● $\overline{LU} \parallel \overline{BE}$

(4) $\overline{LU} \cong \overline{EU}$

1 pair of opposite sides are both $\parallel$ and $\cong$

21 A ladder 20 feet long leans against a building, forming an angle of 71° with the level ground. To the *nearest foot*, how high up the wall of the building does the ladder touch the building?



(1) 15

(3) 18

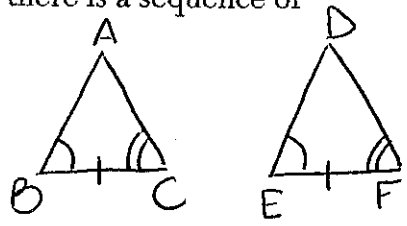
(2) 16

● 19

$\sin 71 = \frac{x}{20}$

$x = 19$

22 In the two distinct acute triangles *ABC* and *DEF*, $\angle B \cong \angle E$. Triangles *ABC* and *DEF* are congruent when there is a sequence of rigid motions that maps



(1) $\angle A$ onto $\angle D$, and $\angle C$ onto $\angle F$ AAA

(2) $\overline{AC}$ onto $\overline{DF}$, and $\overline{BC}$ onto $\overline{EF}$ SSA

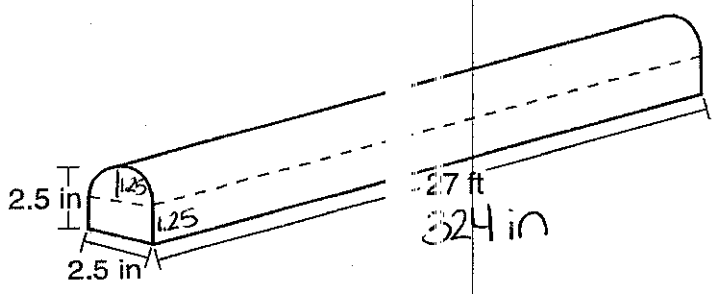
● $\angle C$ onto $\angle F$, and $\overline{BC}$ onto $\overline{EF}$ ASA

(4) point *A* onto point *D*, and $\overline{AB}$ onto $\overline{DE}$

1 ft = 12 in

Use this space for computations.

23 A fabricator is hired to make a 27-foot-long solid metal railing for the stairs at the local library. The railing is modeled by the diagram below. The railing is 2.5 inches high and 2.5 inches wide and is comprised of a rectangular prism and a half-cylinder.



27 ft
324 in

$V_{prism} + \frac{1}{2}(V_{cylinder})$

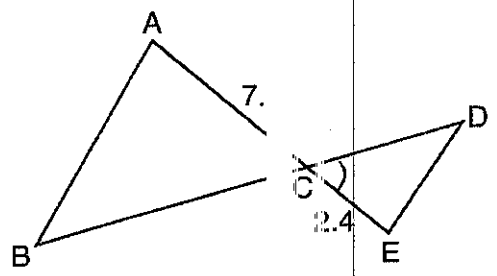
$lwh + \frac{1}{2}\pi r^2 h$

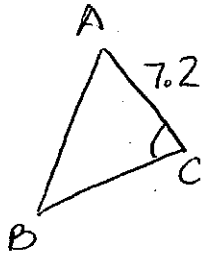
$(2.5)(1.25)(324) + \frac{1}{2}\pi(1.25)^2(324)$

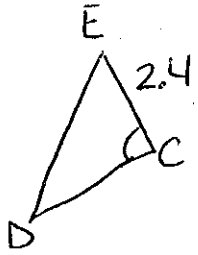
How much metal, to the *nearest cubic inch*, will the railing contain?

(1) 151
(2) 795
(3) 1808
(4) 2025

24 In the diagram below, $AC = 7.2$ and $CE = 2.4$.







Which statement is *not* sufficient to prove $\triangle ABC \sim \triangle EDC$?

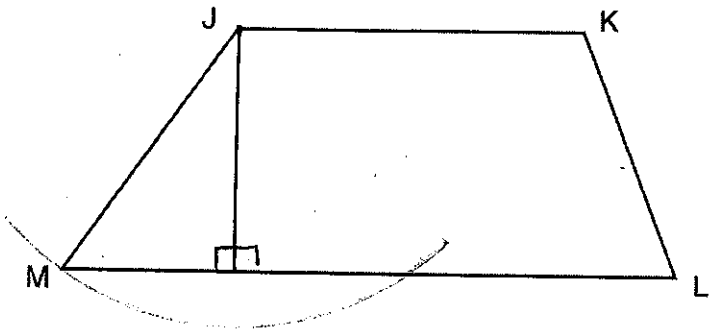
(1) $\overline{AB} \parallel \overline{ED}$ AA
(3) $DE = 2.7$ and $AB = 8.1$ SA
(3) $CD = 3.6$ and $BC = 10.8$ AS
(4) $DE = 3.0$, $AB = 9.0$, $CD = 9$, and $BC = 8.7$ SSS

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Given: Trapezoid $JKLM$ with $\overline{JK} \parallel \overline{ML}$

Using a compass and straightedge, construct the altitude from vertex J to $\overline{ML}$. [Leave all construction marks.]



26 Determine and state, in terms of π, the area of a sector that intercepts a 40° arc of a circle with a radius of 4.5.

A.S. $\frac{n}{360} \cdot \pi r^2$

$\frac{40}{360} \cdot \pi(4.5)^2$

2.25π or $\frac{9\pi}{4}$

27 The diagram below shows two figures. Figure A is a right triangular prism and figure B is an oblique triangular prism. The base of figure A has a height of 5 and a length of 8 and the height of prism A is 14. The base of figure B has a height of 8 and a length of 5 and the height of prism B is 14.

Figure A

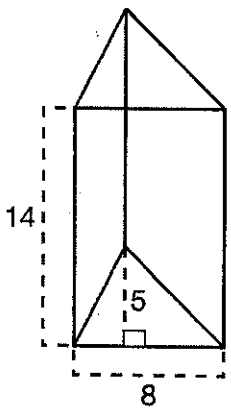
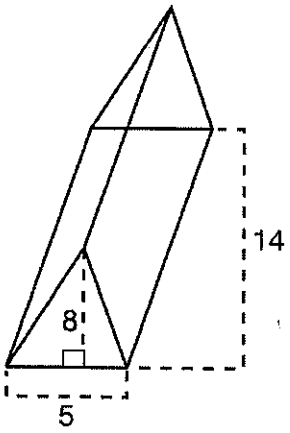


Figure B



Use Cavalieri's Principle to explain why the volumes of these two triangular prisms are equal.

Base Areas $\frac{1}{2}(5)(8) = \frac{1}{2}(8)(5)$

$20 = 20$

Heights $14 = 14$

If the base areas are equal and the heights are equal then Cavalieri's Principle states the volumes will be equal

28 When volleyballs are purchase they are not fully inflated. A partially inflated volleyball can be modeled by a sphere whos volume is approximately 180 in^3. After being fully inflated, its volume is approximately 294 in^3. To the *nearest tenth of an inch*, how much does the radius increase when the volleyball is lly inflated?

$V_{\text{partially inflated sphere}}$

$\frac{180}{1} = \frac{4\pi r^3}{3}$

$\frac{540}{4} = \frac{4\pi r^3}{4}$

$\frac{135}{\pi} = \frac{\pi r^3}{\pi}$

$\sqrt[3]{42.97183463} = \sqrt[3]{r^3}$

$r = 3.5026$

$V_{\text{fully inflated sphere}}$

$\frac{294}{1} = \frac{4\pi r^3}{3}$

$\frac{882}{4} = \frac{4\pi r^3}{4}$

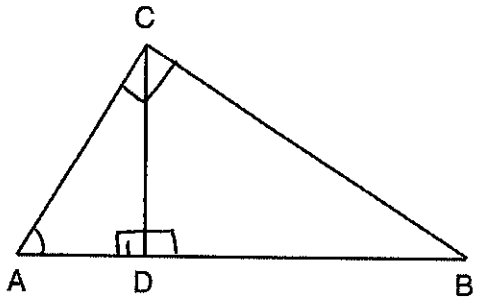
$\frac{220.5}{\pi} = \frac{\pi r^3}{\pi}$

$\sqrt[3]{70.1873299} = \sqrt[3]{r^3}$

$r = 4.1249584406$

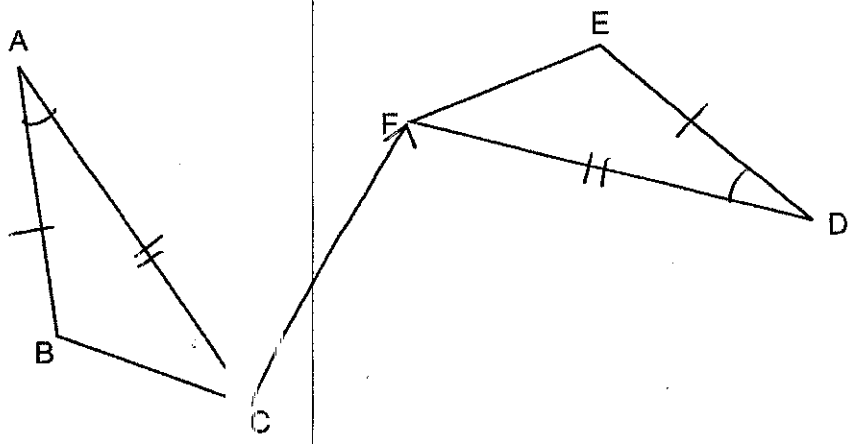
$4.1249584406 - 3.5026 = 0.62$

29 In right triangle ABC shown below, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$. Explain why $\triangle ABC \sim \triangle ACD$.



S	R
① Right ABC, Altitude $\overline{CD}$ drawn to hypotenuse $\overline{AB}$	① Given
② $CD \perp AB$	② An altitude is $\perp$ to base
③ $\angle 1$ is a right $\angle$	③ $\perp$ line form right $\angle$s
④ $\angle C$ is a right $\angle$	④ $\triangle ABC$ is a right $\triangle$ which has a right $\angle$
⑤ $\angle 1 \cong \angle C$	⑤ All right $\angle$s are $\cong$
⑥ $\angle A \cong \angle A$	⑥ Reflexive
⑦ $\triangle ABC \sim \triangle ACD$	⑦ AA $\sim$

30 Triangle ABC and triangle DEF are drawn below.



If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$, write a sequence of transformations that maps triangle ABC onto triangle DEF.

A translation along vector $\overrightarrow{CF}$ so C maps to F
A rotation around point F so that $\angle A$ maps onto $\angle D$, $\overline{AB}$ maps to $\overline{DE}$ and $\overline{AC}$ maps to $\overline{DF}$

31 Line n is represented by the equation $3x + 4y = 20$. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point (4,2). [The use of the set of axes below is optional.]

Explain your answer.

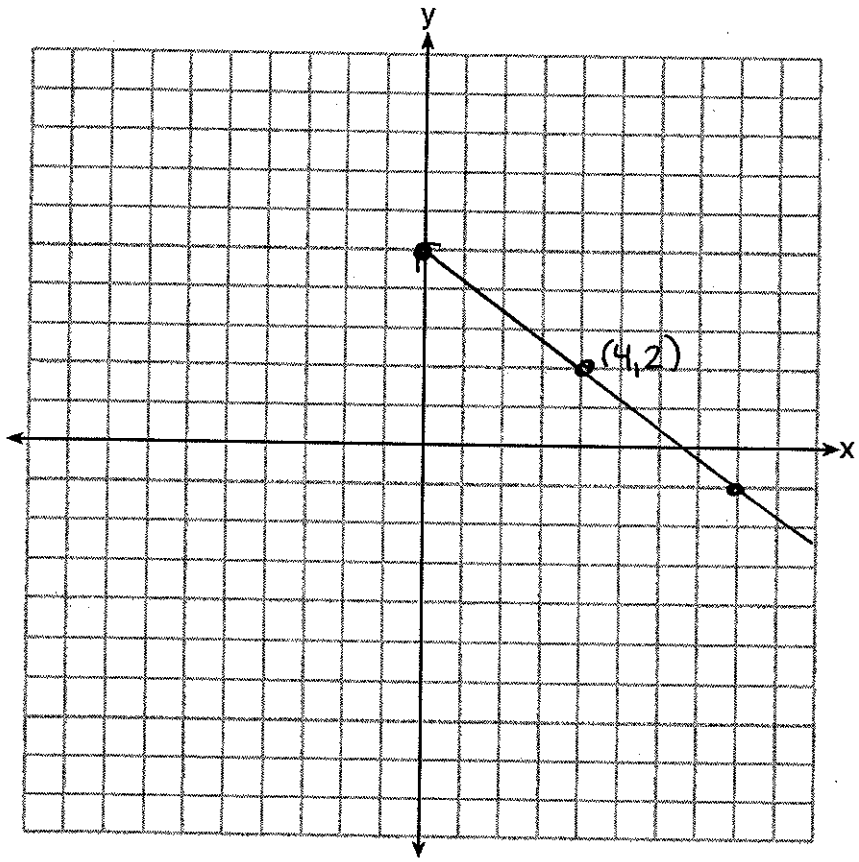
$3x+4y=20$

$-3x \quad -3x$

$\frac{4y}{4} = \frac{-3x+20}{4}$

$y = \frac{-3}{4}x+5$

* since point (4,2) is on the line, the line remains the same



Part III

Answer all 3 questions in thi part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, inch ling appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the informati provided for each question to determine your answer. Note that diagrams are not necess rily drawn to scale. For all questions in this part, a correct numerical answer with no work hown will receive only 1 credit. All answers should be written in pen, except for graphs id drawings, which should be done in pencil. [12]

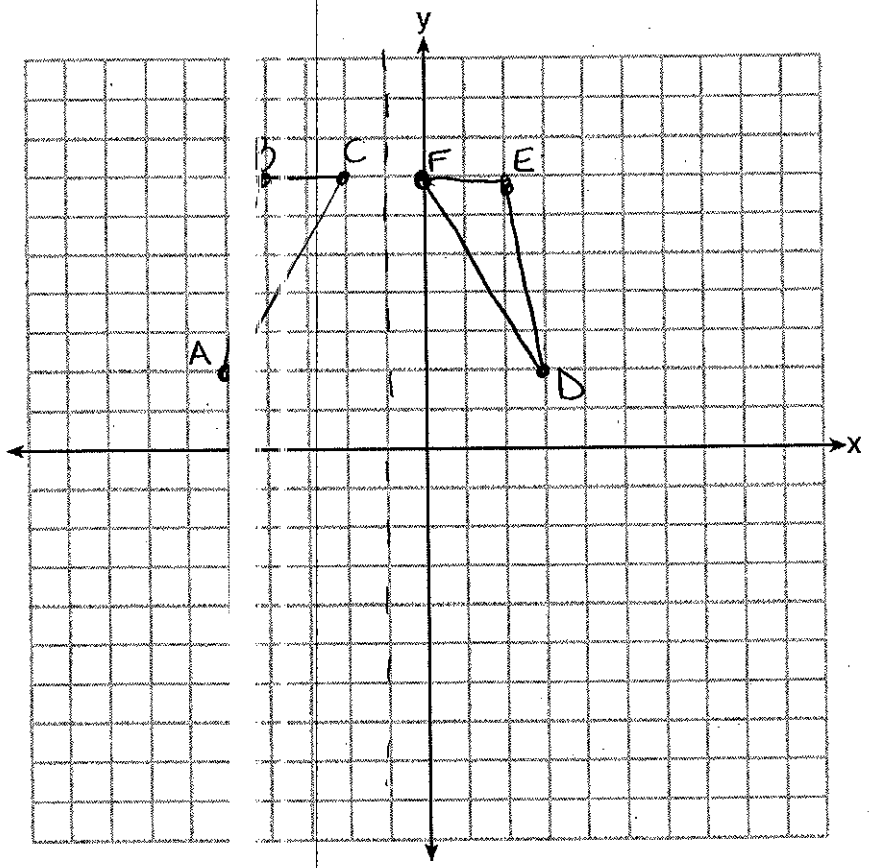
32 Triangle ABC has vertices at $A($ 5,2), $B(-4,7)$, and $C(-2,7)$, and triangle DEF has vertices at $D(3,2)$, $E(2,7)$, and $F(0,7)$. Grap and label $\triangle ABC$ and $\triangle DEF$ on the set of axes below.

Determine and state the single ar sformation where $\triangle DEF$ is the image of $\triangle ABC$.

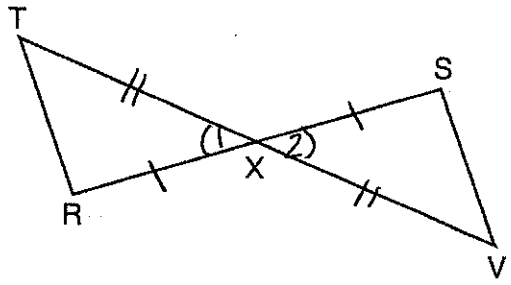
reflection over li ne $x = -1$

Use your transformation to exp in why $\triangle ABC \cong \triangle DEF$.

A line reflection is a rigid motion which preserves distance & angle measure therefore $\triangle ABC \cong \triangle DEF$



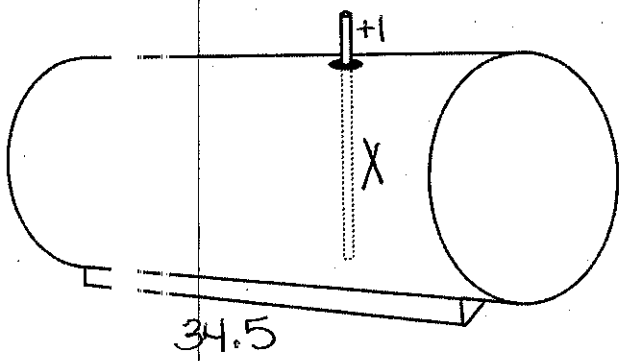
33 Given: $\overline{RS}$ and $\overline{TV}$ bisect each other at point X
$\overline{TR}$ and $\overline{SV}$ are drawn



Prove: $\overline{TR} \parallel \overline{SV}$

S	R
① $\overline{RS}$ & $\overline{TV}$ bisect each other at point X	① Given
② $\overline{RX} \cong \overline{SX}$, $\overline{TX} \cong \overline{VX}$	② Segment bisectors divide a segment into 2 $\cong$ segments
③ $\angle 1 \cong \angle 2$	③ Intersecting lines form $\cong$ vertical $\angle$s
④ $\triangle TXR \cong \triangle VXS$	④ SAS $\cong$ SAS
⑤ $\angle T \cong \angle V$ or $\angle R \cong \angle S$	⑤ CPCTC
⑥ $\overline{TR} \parallel \overline{SV}$	⑥ If alternate interior $\angle$s are $\cong$ then the lines cut by the transversal are $\parallel$

34 A gas station has a cylindrical fue ng tank that holds the gasoline for its pumps, as modeled below. The tank holds a maximum of 2 000 gallons of gasoline and has a length of 34.5 feet.



A metal pole is used to measu how much gas is in the tank. To the *nearest tenth of a foot*, how long does the pole need tc e in order to reach the bottom of the tank and still extend one fc ot outside the tank? Justify yc r answer. [1 ft^3 = 7.48 gallons]

$\frac{1\ ft^3}{X} = \frac{7.48\ g}{20000\ g}$

$\frac{7.48X}{7.48} = \frac{20000}{7.48}$

$X = 2673.796791$

$V = \pi r^2 h$

$\frac{2673.796791}{34.5} = \frac{\pi r^2 (34.5)}{34.5}$

$\frac{77.50135627}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{24.66944789} = \sqrt{r^2}$

$4.966834796 = r$

$X \quad 2$

$9.933669593 = D$

$+ \ 1$

$\boxed{10.9}$

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$.

Prove that $PQRS$ is a rhombus. 4 ≅ sides
[The use of the set of axes on the next page is optional.]

$PQ = \sqrt{(-2-3)^2+(3-8)^2}$

$\sqrt{(-5)^2+(-5)^2}$

$\sqrt{25+25}$

$\sqrt{50}$

$QR = \sqrt{(3-4)^2+(8-1)^2}$

$\sqrt{(-1)^2+(7)^2}$

$\sqrt{1+49}$

$\sqrt{50}$

$RS = \sqrt{(4-1)^2+(1-4)^2}$

$\sqrt{(5)^2+(5)^2}$

$\sqrt{25+25}$

$\sqrt{50}$

$SP = \sqrt{(-1-2)^2+(-4-3)^2}$

$\sqrt{(1)^2+(-7)^2}$

$\sqrt{1+49}$

$\sqrt{50}$

Quad PQRS is a rhombus b/c all sides are ≅

Question 35 is continued on the next page.

Question 35 continued.

Prove that *PQRS* is *not* a squar
[The use of the set of axes belc is optional.]

$PR = \sqrt{(-2-4)^2+(3-1)^2}$

$\sqrt{(-6)^2+(2)^2}$

$\sqrt{36+4}$

$\sqrt{40}$

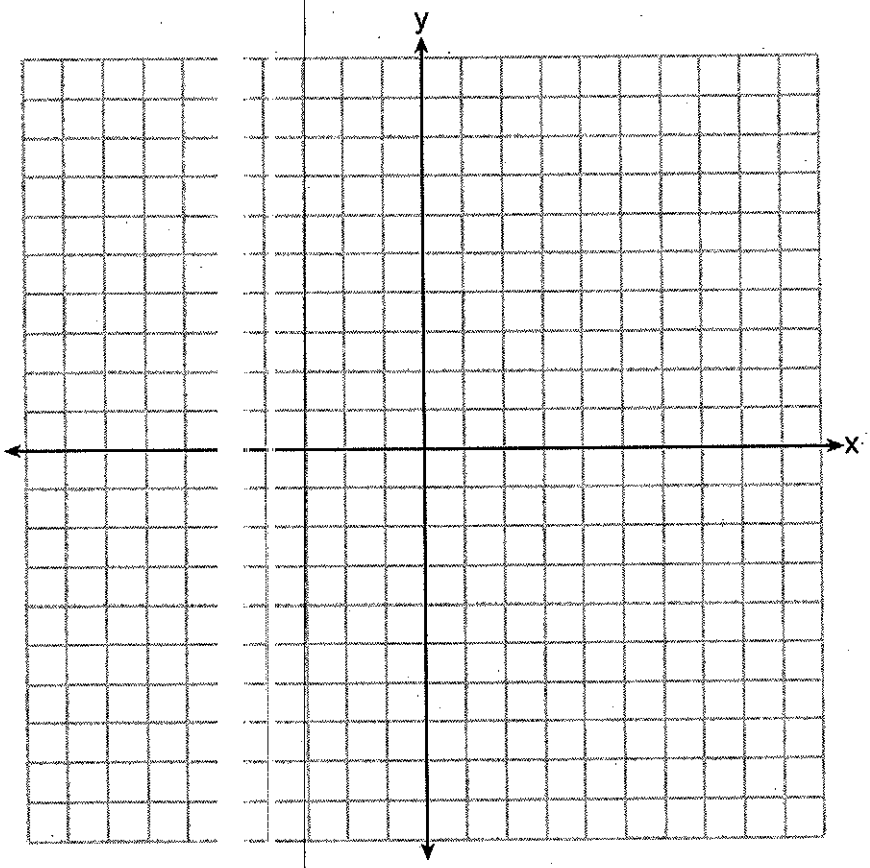
$QS = \sqrt{(3-1)^2+(8-4)^2}$

$\sqrt{(4)^2+(12)^2}$

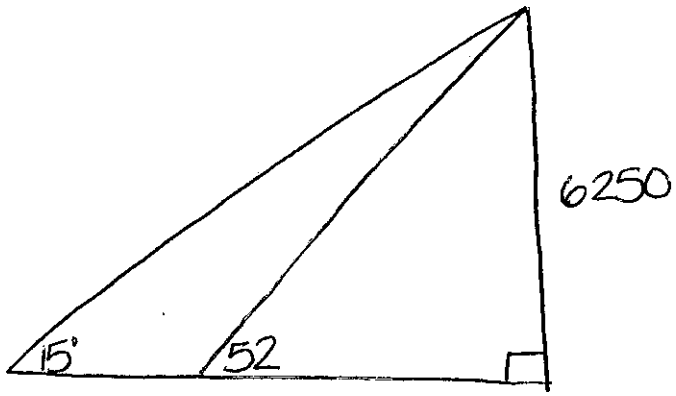
$\sqrt{16+144}$

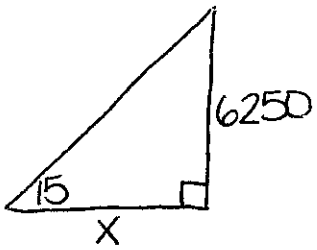
$\sqrt{160}$

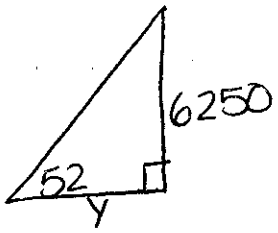
Rhombus PQRS is not a square b/c the diagonals are nc $\cong$



36 Freda, who is training to use a radar system, detects an airplane flying at a constant speed and heading in a straight line to pass directly over her location. She sees the airplane at an angle of elevation of 15° and notes that it is maintaining a constant altitude of 6250 feet. One minute later, she sees the airplane at an angle of elevation of 52°. How far has the airplane traveled, to the *nearest foot*?







Determine and state the speed of the airplane, to the *nearest mile per hour*.

$\tan 15 = \frac{6250}{X}$

$X \frac{\tan 15}{\tan 15} = \frac{6250}{\tan 15}$

$X = 23325.31755$

$\tan 52 = \frac{6250}{Y}$

$Y \frac{\tan 52}{\tan 52} = \frac{6250}{\tan 52}$

$Y = 4883.035166$

$23325.31755 - 4883.035166 = 18442.28238$

18442 ft

$\frac{1 \text{ mile}}{X \text{ mile}} = \frac{5280 \text{ ft}}{18442 \text{ ft}}$

$\frac{5280X}{5280} = \frac{18442}{5280}$

$X = 3.49280303 (60) = 210$